Depletion in a Medical Sterilizer

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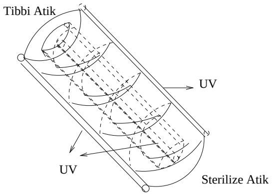
Outline

1. Problem description
2. Auxiliary Problem(Model and analysis)
3. Cylinderical Problem(Model, numerical method and analysis)
4. Conclusions(tips for design)

A Medical Sterilizer



Auxiliary Problem

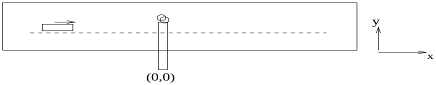


Figure: A single UV on a straight path

Model

- $c(x, t; y)$ concentration at position x and time t with a fixed vertical distance y.
- Beer-Lambert absorption law $Q(x; y) = e^{-\sqrt{x^2+y^2}}$.
- Assumption: In an environment that moves with a velocity u_0, the concentration $c(x, t; y)$ decreases at the rate $Q(x; y)$ for each constant y.
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$$\begin{aligned} \frac{\partial c(x,t;y)}{\partial t} + u_0 \frac{\partial c(x,t;y)}{\partial x} &= -Q(x;y)c(x,t;y), \qquad (1) \\ c(x,0;y) &= 1 \end{aligned}$$

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Solution

$$c(x,t;y) = e^{-\frac{1}{u_0}\int_{x-u_0 t}^{x} Q(s;y)ds}. \tag{2}$$

On the path $\xi = x - u_0 t =$const. $\frac{dc(\xi;y)}{dt} \leq 0$.

Properties

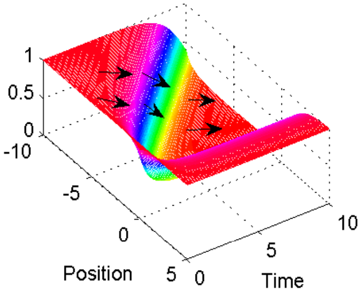

Figure: Depletion for $u_0 = 1$(left) and $u_0 = 2$(right).

Properties

- the farther to the left of the UV source, the longer it takes for depletion to start, as the waste moves from left to right with a constant velocity $u_0 > 0$,
- depletion starts sooner with larger u_0, as the corresponding trajectories reach the close neighborhood of the UV source first,
- depletion is larger for smaller u_0(compare the z axis scales), as the corresponding trajectories get more benefit of the UV light, and
- the waste with initial positions to the right of the UV source, $X = 0$, gets very little benefit of UV light, and thus almost no depletion.

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UV sources on a straight path

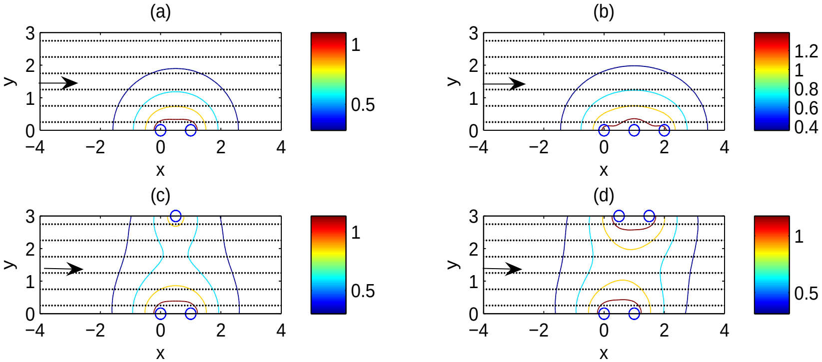
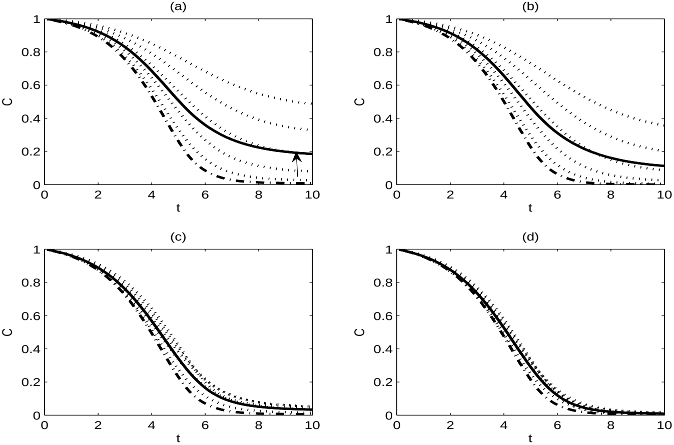


Figure: UV lights of various number and location sheding light on waste sample trajectories(y=const).
$(x, y) = (0, 0), (1, 0)$(Figure(a));$(x, y) = (0, 0), (1, 0), (2, 0)$(Figure(b)),
$(x, y) = (0, 0), (1, 0), (1/2, 3)$,(Figure(c)) and
$(x, y) = (0, 0), (1, 0), (1/2, 3), (3/2, 3)$(Figure((d)).

Depletion along a path with multiple UV light sources.



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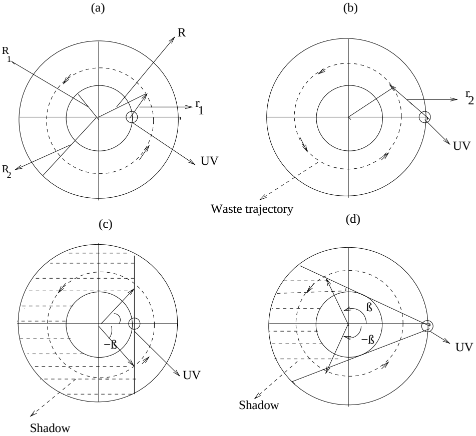
Crossection of sterilizer



Absorption

For the inner UV, the angle $\beta = \cos^{-1}(R_1/R)$ so, ϕ ranges in $[-\cos^{-1}(R_1/R), \cos^{-1}(R_1/R)]$. For outer UV, $\beta = \pi - (\sin^{-1}(R_1/R_2) + \sin^{-1}(R_1/R))$ so, ϕ ranges in $[-\beta, \beta]$.
Absorption along the helical path

$$Q(r) = \begin{cases} e^{-r}, & \text{eger } -\beta \leq \theta - 2\pi k \leq \beta \\ 0 & \text{otherwise} \end{cases} \quad k = 0, 1, \ldots \tag{3}$$

where $r = r_1$ inner UV(Figure(a)) and $r = r_2$ outer UV(Figure(b)), $r_1 = \sqrt{R_1^2 + R^2 - 2R_1 R \cos\theta}$ and $r_2 = \sqrt{R_2^2 + R^2 - 2R_2 R \cos\theta}$.

Model

Let $c(\theta, t; r)$ represent depletion at (r, θ) and time t.

$$\begin{aligned} \frac{\partial c(\theta, t; R)}{\partial t} + v \frac{1}{R} \frac{\partial c(\theta, t; R)}{\partial \theta} &= -Q(r)c(\theta, t; R), t > 0 \\ c(\theta, 0; R) &= 1. \end{aligned} \tag{4}$$

We assume that medical waste goes through a rigid body motion along a path with radius R,so $v = wR$, where w is the angular velocity.

Solution

For the first cycle where $\theta \in [-\beta, \beta]$ the solution becomes

$$c(\theta, t; R) = e^{-\frac{1}{w}\int_{\theta-wt}^{\theta} Q(s;R)ds}. \tag{5}$$

Outside the interval $[-\beta, \beta]$ $c(\theta, t; R)$ attains different constant values. We are interested in depletion over the trajectories $\xi = \theta - u_0 t = \theta(0) = [\theta_1(0), \theta_2(0), \ldots, \theta_m(0)]$, where $\theta(0)$ consists of the waste positions before the exposure. On the trajectories $\xi = const$, equation (4) becomes

$$\begin{aligned} \frac{dc_{i,j}}{dt} &= -Q(\theta_i(t), \mathbf{R}(j))c_{ij} \\ c_{i,j}(0) &= c(\theta_i(0), \mathbf{R}(j)) = 1, i = 1, 2, \ldots, m; j = 1, \ldots, n \end{aligned} \tag{6}$$

where $\mathbf{R}$ is the vector of trailing radii in the interval (R_1, R_2) for n pieces of waste.

Definitions

We define

$$\overline{c}(t) = \frac{1}{mn} \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}(t)$$

as the averaged depletion at t,

$$D = \int_0^T \overline{c}(t)dt$$

as the depletion over the time interval $[0, T]$.

Numerical method

If we define
$\mathbf{C}(t) = [c_{i,j}(t)], \mathbf{Q}(t) = [Q(\theta_i(t), \mathbf{R}(j))], i = 1, 2, \ldots, m; j = 1, \ldots, n$ then system (6) can be written as a matrix-valued system of differential equations

$$\begin{aligned} \frac{d\mathbf{C}}{dt} &= -\mathbf{Q}. * \mathbf{C} \\ \mathbf{C}(0) &= \mathbf{1}, \end{aligned} \tag{7}$$

where $\mathbf{1}$ is of course a matrix of size $m \times n$,with all entries are equal to 1 and $'.*'$ is the entry-wise multiplication as used by MATLAB.

Experiment I(The effect of initial position)

$R_1 = 1$, and trailing radius $R = 1.1$. UV source is located at the position $(r, \theta) = (R_1, 0)$, and we let $w = 2$. We observe depletion on waste with initial positions $\theta_0 = [-\pi, -\pi/2, 0, \pi/2]$.Depletion curves $\theta = \theta_0$ are displayed in Figure 6 with dotted curves, while their average, $\overline{c}(t)$ is the solid curve.

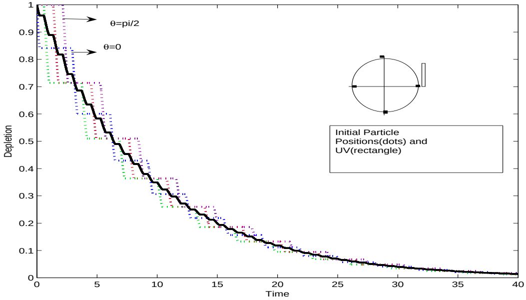


Figure: Depletion curves(dotted) with different starting points and their average(solid).

Experiment II (The effect of angular speed, averaged over initial points)

$(r, \theta) = (1, 0)$

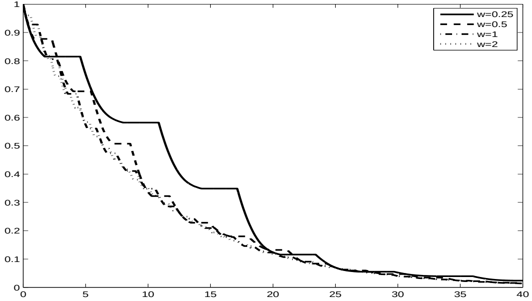


Figure: Depletion on various trailing radius.

- the larger the angular speed w, the smaller the jumps in the depletion curve

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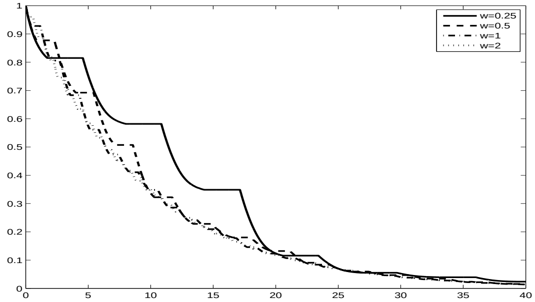

Figure: Depletion on various trailing radius.

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Experiment III(The effect of trailing radius)

Averaged over initial points $\theta(0) = -\pi : \pi/10 : (9\pi/10)$ and trailing radii $R = (R1 + 0.1) : 0.5 : R2$, where $R1 = 1, R2 = 4$, UV $(R_1, 0), w = 6$. The thickest curve $R = 1.1$.

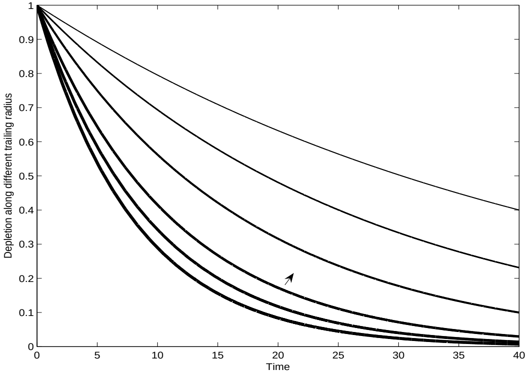


Figure: Depletion with various trailing radius

Experiment IV(Effect of outer radius)

We let $R_1 = 1$ and take $R_2 = 1.2 : 0.1 : 3$. We place a UV at $(R_1, 0)$. For each R_2, we consider the waste trailing on the radii $R = R_1 : 0.1 : R_2$. For each R, we consider the trajectories with initial starting points $\theta(0) = -\pi : \pi/10 : 9\pi/10$. The resulting averaged values are displayed in the Figure for $w = [0.5\ 0.6\ 0.71\ 1.52\ ]$,

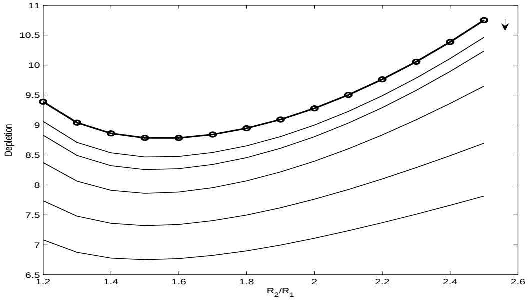
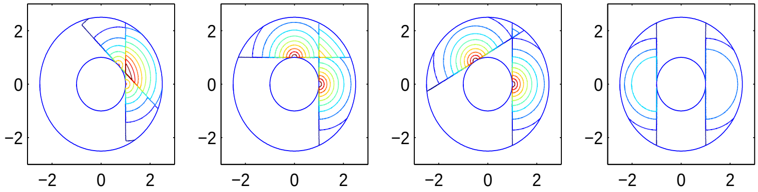


Figure: Depletion versus R_2/R_1 with various w's.

Absorption due to two UV sources as one changes position



Experiment V(optimal position for the second UV)

We place the first UV at $(r, \theta) = (1, 0)$ and the second at $(1, I)$,
$I = [-\pi : \pi/20 : \pi]$,
$\theta_0 = -\pi : \pi/10 : 9\pi/10$ and $R = (R_1 + 0.1) : 0.1 : R_2$
$R_2 = 1.6$(left) and $R_2 = 2$(right), $w = 1$. The dotted curves in UV(II) correspond to positions that are not optimal. Optimal $\theta = \pm 130$

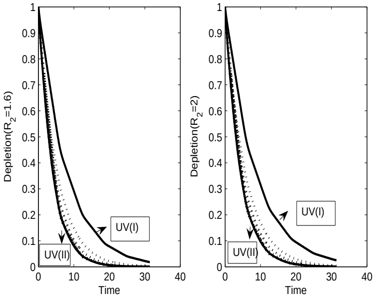


Figure: A single UV versus two UV lights

Exit concentration values

Residence time T=31.4159, $L = 2$, device length, $b = 0.4$,axial distance between helical paths; $T = 2\pi L/(wb))$

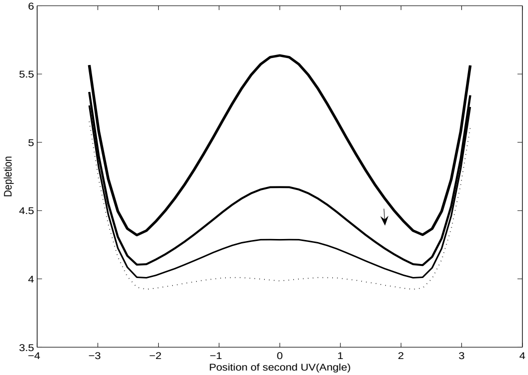
Table I: Exit concentrations and their ratios

R_2	UV(I) Exit values	UV(II) Exit values	UV(I)/UV(II)
1.6	0.0179	3.5712e-004	50.1232
2	0.0248	7.8075e-004	31.7643

- The gain is much larger than linearly expected value of 2.

Optimal angle

Depletion averaged over the $\theta_0's$ and the trailing radius with various angular speeds w =[0.5,0.75,1,1.5] for $R_2 = 1.6$. We see that the best position for the second UV light for this w values are [$\pm$135,$\pm$131,$\pm$131,$\pm$128] which yield the minimal depletion over the time interval $[0, T]$ as in Experiment V.



Two UV sources(A (UV_{in}) and a (UV_{out}))

Absorption due to a UV_{in} and a UV_{out}

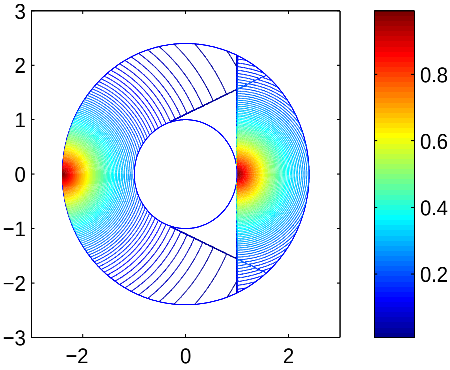


Figure: A UV_{in} at $(R_1, 0)$ and a UV_{out} at (R_2, π)

Experiment VI (A UV_{out} vs UV_{in})

We investigate the effect of $\mathbf{UV}_{out}$ versus $\mathbf{UV}_{in}$ for various R_2 We place a UV_{in} at $(R_1, 0)$, vary θ_0 values as before and compute depletion for each UV_{out} at $(R_2, 0)$, $R_2 = 1.25, 1.75.(R_1 = 1)$.

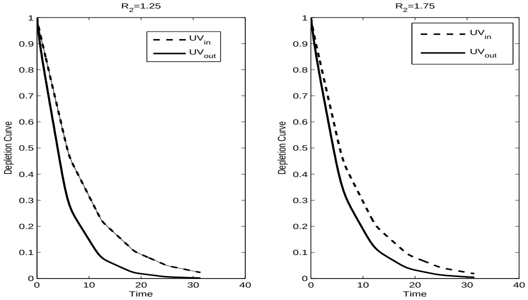


Figure: Depletion curves for UV_{in} and UV_{out}

From the Figure, we observe that UV_{out} leads to better depletion for both

Exit concentrations and Depletion

Table 2:Exit concentrations and averaged depletion

R_1/R_2	0.8	0.57
UV_{in} Exit concentration	0.0233	0.0187
UV_{out}	0.018	0.0046
Ratio	1.2944	4.0652
Depletion UV_{in}	8.37	7.9535
Depletion UV_{out}	5.24	5.9543
Ratio	1.5973	1.3358

Experiment VII(The effect of a **UV**$_{out}$ and a **UV**$_{in}$)

We place a UV$_{out}$ at $(R_2, 0)$ and determine optimal position for UV$_{in}$, where $R_1 = 1$ and R_2 values are as indicated in Table 3.

Tablo 3:Optimal angles and depletion

R2	Angle	Depletion
1.2	± 140	3.2155
1.25	± 140	3.2064
1.3	± 140	3.1255
1.4	± 137.5	3.1264
1.5	± 132.5	3.1770
1.6	± 132.5	3.2581
1.75	± 130	3.4193

Design would not be optimal with two lights where $R_2 > 1.4$. Optimal pair in this case is $(1, 1.3)$

Depletion

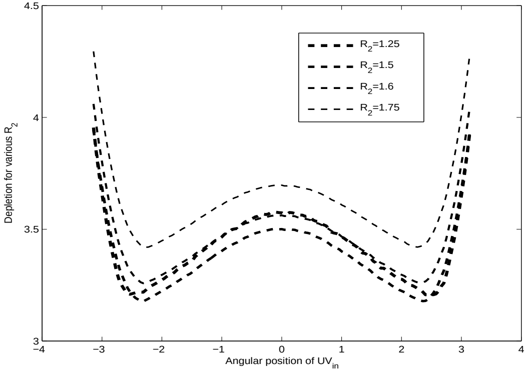


Figure: Depletion versus UV_{in} positions for various R_2.

Experiment VIII(The effect of a $\mathbf{UV}_{out}$ and a combination of two $\mathbf{UV}_{in}$ s.

For the same R_2 values as in Table 4, the optimal position of UV_{in} and depletion.

Tablo 4:Optimal angles and depletion

R_2/R_1	Angle($\mathrm{UV}_{in}(I)$),Angle($\mathrm{UV}_{in}(II)$)	Depletion
1.2	140,-130	2.2783
1.25	140, -130	2.2741
1.3	140 ,-130	2.2059
1.4	130 ,-130	2.1921
1.5	130, -130	2.2119
1.6	120, -130	2.2548
1.75	120, -120	2.3428

Conclusions

- We have proposed a convection-reaction model to investigate medical waste depletion through UV lights. By changing the control parameters, such as the inner to outer radius ratio, rotational speed, number of UV lights and their position, we have computed averaged depletion curves over the random parameters, namely the initial position of particles in the device, and their trailing radius. We have observed that
- the depletion curve on a constant speed waste particle is a piecewise smooth curve that tends to zero exponentially as $t \to \infty$ (Experiment I),
- higher rotational speeds force depletion curves with different initial positions to come closer, which implies that initial positions are less important(Experiment II),
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Sonuçlar

- The optimal position for two UV's along the same cylinder are around, ± 130 degrees with respect to each other(Experiment V),
- an outer UV is much more effective than the inner one(Experiment VI),
- in case of using a UV_{out} and a UV_{in}, the optimal positions for the UV_{in} in the neighborhood of 140 degrees as illustrated in Table 3 with optimal $R_2/R_1 = 1.3$. In case of using a UV_{out} and two UV_{in}, the optimal positions for the two UV_{in} 's are in the neighborhood of (130,-130) with optimal $R_2/R_1 = 1.4$ as illustrated in Table 4(Experiment VII).
- the numerical procedure used here can be implemented for similar time-dependent multi-particle convection-reaction systems.

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Thanks

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- Leave of absence by Karadeniz Technical University,
- The funding provided by Turkish Higher Education Committee.

Kaynakça

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