Numerical Analysis

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- Analytical

Mathematical Analysis

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- Numerical

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- Qualitative

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- Analytical
- Numerical
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- Symbolic

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$$y' = t - y, t \in (a, b), y(a) = y_0?$$

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$$a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

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$$\begin{aligned} y\prime &= t - y^2, t \in (a, b) \\ y(a) &= y_0 \end{aligned}$$

Qualitative analysis

- Learning about the qualitative behaviour of solution without solving the equation itself. Consider for example,

$$\begin{aligned} y' &= y(1-y) \\ y(0) &= y_0 \end{aligned}$$

If $y_0 > 1$ then RHS is negative so $y\prime < 0$. Thus, solution curves, having netative slopes, will tend to the asymptote $y = 1$, as $t \to \infty$

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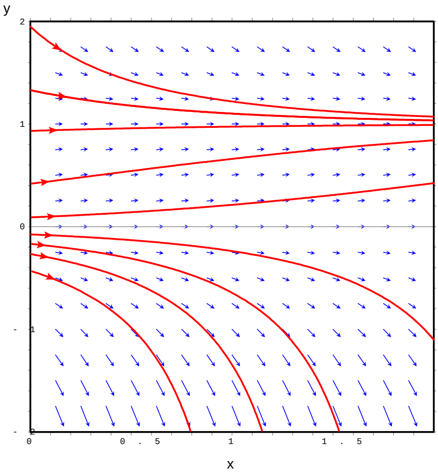
- On the other hand if $y_0 < 0$ then RHS is negative so it is clear even without having explicit solution that the solution curves will decrease.

- plotdf function of `Maxima` verifies our predictions of solution curves.



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- Let's consider the following initial value problem and see how its analytical solution is obtained by Maxima.

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- $$\begin{aligned} y'' + y' &= x \\ y(0) &= 0, y'(0) = 0 \end{aligned}$$

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```
(%i2) denk:'diff(y,x,2)+'diff(y,x)=x;
```

(%o2) $\frac{d^2}{d\,x^2}\,y + \frac{d}{d\,x}\,y = x$

```
(%i3) ode2(denk,y,x);
```

(%o3) $y = \%k2\,\%e^{-x} + \frac{x^2 - 2\,x + 2}{2} + \%k1$

```
(%i4) ic2(%,x=0,y=0,'diff(y,x)=0);
```

(%o4) $y = \frac{x^2 - 2\,x + 2}{2} - \%e^{-x}$

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1. begins with a properly formulated mathematical `problem`,

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1. begins with a properly formulated mathematical `problem`,
2. numerical method of solution,
3. algorithm to implement the numerical method in an electronic platform,
4. code of algorithm with an apropriate programming language,
5. testing of code,
6. results, interpretations, critics of numerical method and search for alternatives

Stages of numerical analysis(Example-I:Determining interval containing a zero of function)

- **Problem**:Determine an interval $[a, b]$ containing zero of a function, if exists, near a given point x_0.

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- **Problem**:Determine an interval $[a, b]$ containing zero of a function, if exists, near a given point x_0.
- **Numerical method(**search along right or left directions**):** begin with interval $[x\min, x\max] := [x_0 - R, x_0 + R], R > 0$ *sabit*, and call it as a zero search interval. Starting with $x = x_0$, search towards right at the points

$$x, x + h, x + 2h, ...$$

until capturing the first interval $(x, x + h)$ for which

$$f(x)f(x + h) <= 0$$

In this case the required interval is $X = [x, x + h]$.

Stages of numerical analysis(Example-I:Determining interval containing a zero of function)

- In case a required interval could not be found through searching along the direction towards the right, start with $x = x_0$ and search through the points,

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- If left or right direction search does not result in a proper interval, then no zero is found in the search interval $[x\min, x\max]$.

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  - receiving relevant data from user, called input
  - ordered set of instructions to implement the associated numerical method
  - returning relevant data to user, called output

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7. display "no interval has been determined " set $X = []$ and return.

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7. display "no interval has been determined " set $X = []$ and return.
8. `Output:` X

Example-I:Program(or Code)

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- $while\ x > x\min$

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- *while* $x < x\max$
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- $else$
- $x = x - h;\ end$
- end
- $disp('\text{no interval has been determined}'); X = [];$

Example-I:Test and implementation

- Determine an interval $[a, b]$ of length $h = 0.1$ that contains a zero of $f(x) = exp(x) - x - 4$ near $x_0 = 0$

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- ```
  >> f=@(x) exp(x)-x-4

  >> X=bul(f,0)
  X= 1.7000 1.8000
  ```
  ```

Example-ITest

- Determine an interval $[a, b]$ of length $h = 0.1$ that contains a zero of $f(x) = log(x) - x + 4$ near $x_0 = 10$

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  >> X=bul(f,10)
  X=5.7000 5.8000
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Example-I: Restrictions and alternatives

- Intervals around discontinuity where the function changes sign may mistakenly be considered as an interval containing zero

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- The method can be generalized to handle such situations.

Stages of numerical analysis(Example-II)

- **Problem(determinig real zero of functon)**:Let f be a continuous function that changes sign over the end points of $[a, b]$, *that is,* $(f(a)f(b) < 0)$ *.Determine an approximation for the zero of* f *over the interval* $[a, b]$ *.*

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- Solution to the problem exists due to the `Intermediate Value Theorem` for continuous functions.

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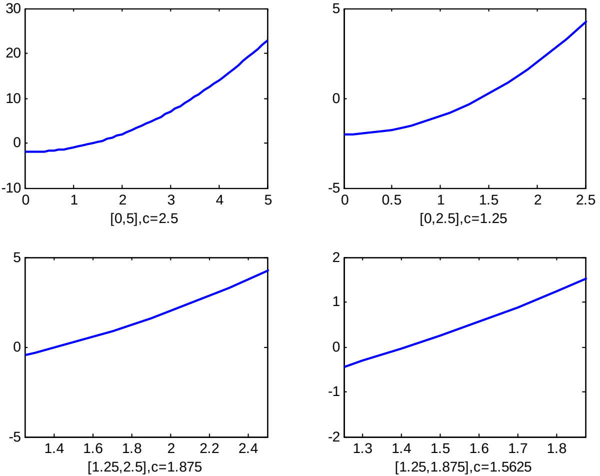

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2. $c = (a+b)/2$
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4. while $|f(c)| > \epsilon$ writedown $a, c, b, f(c)$ and goto (2) else return c as the final approximation.

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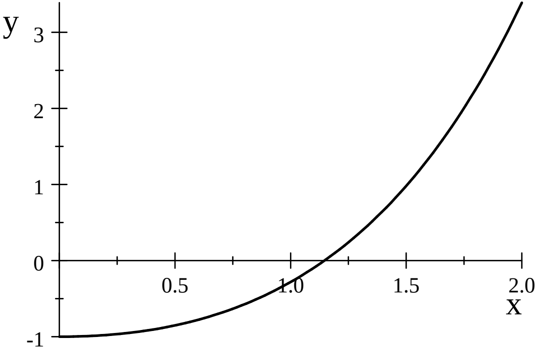

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- Approximation for zero accurate to 15 decimal digits is $c = 1.146193220620583$.

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Teorem 1

Let f be a function that changes sign over the end points of the interval $[a, b] = [a_1, b_1]$, r be the zero of f within the interval $[a_n, b_n]$ with $f(a_n)f(b_n) < 0$ and $\{c_n\}_{n=1}^{\infty}$ be the sequence of midpoints, that is, $c_n = (a_n + b_n)/2$. Then

$$\lim_{n\to\infty} c_n = r$$

Example-II:Convergence analysis

Proof: For the zero r, at the n-th iteration we have

$$|r - c_n| \leq \frac{b_n - a_n}{2}$$

On the other hand, since the length of each subinterval is half of the previous one, we have

$$|r - c_n| \leq \frac{b_n - a_n}{2} = \frac{b_{n-1} - a_{n-1}}{2^2} = \cdots = \frac{b_1 - a_1}{2^n}$$

from which we have $\lim_{n\to\infty} |r - c_n| = 0$. The result follows from the inequality

$$-|r - c_n| \leq r - c_n \leq |r - c_n|$$

along with sandwitch theorem [7] .

Example-II:Convergence analysis

Tanım 1

Let $\{c_n\}_{n=0}^{\infty}$ *be a sequence that converges to point* r. *If there exists a positive integer* N *such that the inequality*

$$|r - c_{n+1}| \leq \alpha |r - c_n|^{\beta}$$

holds for all $n \geq N$ *with* $\alpha > 0, \beta \geq 1$,*then we say that the sequence* $\{c_n\}_{n=1}^{\infty}$ *is convergent of order* β. *If* $\beta = 1$ *then for convergence we need to have* $\alpha \in (0, 1)$,*in this case the squence is said to be linearly convergent. If* $\beta = 2$ *then the squence is said to be quadratically convergent.*

Example-II:Convergence analysis

- For the bisection method, comparing the inequality

$$|r - c_{n+1}| \leq \frac{b_{n+1} - a_{n+1}}{2} = \frac{1}{2}\frac{b_n - a_n}{2}$$

with

$$|r - c_n| \leq \frac{b_n - a_n}{2}$$

we have

$$|r - c_{n+1}| \cong \frac{1}{2}|r - c_n|$$

Then the method is linearly convergent. The approximate ratio $|r - c_{n+1}|/|r - c_n| \cong \frac{1}{2}$ is referred to be a mean convergence rate.

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- prepare a code called **secants** to implement this method
- Note the difference between **divbysecants** and **secants**

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- The method outlined above is known as `Muller's method`([6]).

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Example-III Algorithm(detemining zero of a function near a given point)

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     else use the method of Example 2 with `c=ikibol(f,a,b) and determine c`

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1. `Input f(continuous),x0`
2. Determine an interval $[a, b]$ containing zero of f using the method of Example I.
3. - if $f(a) = 0$ then $c = a$,
     elseif $f(b) = 0$ then $c = b$,
     else use the method of Example 2 with `c=ikibol(f,a,b) and determine c`
4. `Output: c`

Example-III: Program(Code)(detemining zero of a function near a given point)

- `function c=fsifir(f,x0);`

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- $if\ \ isempty(X)$

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- $\quad\quad\quad c = ikibol(f, a, b);$
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Example-III: Test

- Determine the zero of $f(x) = \ln(x) - x + 4$ near $x_0 = 4$

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-
```
>> f=@(x) log(x)-x+4

>> fsifir(f,4)
ans =
5.7490
```

Example-III :Test

- The approximate zero can also be determined using `fzero` `function of MATLAB/OCTAVE fzero`:

  ```
  >> fzero(f,4)
  ans =
  5.7490
  ```

Example-III :Test

- Determine the zero of $f(x) = xsin(1/x)$ near $x_0 = 4$.

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- Determine the zero of $f(x) = xsin(1/x)$ near $x_0 = 4$.
-
```
>> f=@(x) x*sin(1/x)

>> fsifir(f,4)
ans =
0.3183
```

Exercises

1 Given

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$$

carry out numerical analysis of solving linear algebraic system $AX = b$ by the following steps

1. If $det(A) \neq 0$ then use Cramer's method. If $det(A) = 0$ then give a message to user saying that the method can't be implemented. .
2. Prepare an algorithm for the method described in (a). The inputs should be the matrix $A_{2\times 2}$ and vector $b_{2\times 1}$ and output is the solution vector $X_{2\times 1}$.
3. Prepare a code for the algorithm in MATLAB/OCTAVE.
4. Test your code with various coefficient matrices A and right hand side vectors b.

Exercises

2 Given the quadratic equation

$$ax^2 + bx + c = 0$$

carry out numerical analysis of solving the equation by the following steps

1. If $a \neq 0$ then use the well known formulas for the roots, otherwise give a message to the user that it is not a quadratic equation..
2. Prepare an algorithm for the method described in (a). Inputs should be the coeeficients a, b, c , and the outputs the roots(real or complex)
3. Prepare a code for the algorithm in MATLAB/OCTAVE.
4. Programınızı farklı a, b, c katsayıları için test yapınız.
5. Make sure to handle the case $a = 0$, as well.

Exercises

3 Following your analysis of question #2, carry out numerical analysis of determining eigenvalues of a given matrix $A_{2\times 2}$.

4 Intersection point or points(if any) of a circle with radius r and a parabola can be determined by solving the nonlinear algebraic system

$$\begin{aligned} x^2 + y^2 &= r^2 \\ -ax^2 + y &= 0 \end{aligned}$$

Carry out numerical analysis of solving this system. Inputs should be r and $a \neq 0$ and outputs the real solution(s) if any.

5 Given $f(x) = x^2 - 5x$, $a = -2, b = 3$. Determine the first three approximations for zero of f over the interval $[a, b]$ using method of

1. bisection,
2. division by secants and
3. secants(Note that for the case of secants, a and b are the initial approximations, not the end points of the interval $[a, b]$)

Exercises

6 Given $f(x) = x^3 - x - 1,\ a = -2, b = 3$. Determine the first three approximations for zero of f over the interval $[a, b]$ using method of

1. bisection,
2. division by secants and
3. secants

7 By running the code "ikibol" for the following functions and intervals $[a, b]$ determine a zero of each. Take $eps = 1e - 10$.

1. $f(x) = x^2 - 5x,\ a = -2, b = 3$
2. $f(x) = x^3 - x - 1,\ a = -2, b = 3$
3. $f(x) = ln(x + 1) - \frac{1}{4}x^2,\ a = 1, b = 4$
4. $f(x) = 5e^{-x} - coshx,\ a = 0, b = 4$

8. Repeat question #7 for the method of division by secants.
9. Repeat question #7 for the method of secants.
10. Develop an application with inputs f , a,b , eps, and *method* and output a zero of f. The input parameter "method" will take on a value of 1(for bisection),2(for division by secants), or 3(for the method of secants).

Exercises

11 For the functi ons defined in question #7, take $x_0 = a$ (given in the same question) determine an interval containing a zero of f.

12 Use the code *fsifir* given above and fzero of MATLAB/OCTAVE to determine a zero of the functions defined in question 7. Take the a values therein as the initial guess.

13 The roots function of MATLAB/OCTAVE determines all zeros of polynomials with given coeficients.For example the command $>> roots([a\ b\ c])$ determines all the roots of the polynomial

$$p(x) = ax^2 + bx + c$$

Test question 7(a),(b) using roots function.

14 Show that the $x-$intercept points $\{c_n\}$ of secant line $(a_n, f(a_n)), (b_n, f(b_n))$ as used by the method of division by secants converges to a zero of f.

15 Test your computer's `perfommance of numerical analysis` using the following typical problems:

- Check the largest size of matrix you can produce using the command `>>A=rand(n)` with

$$n = 2000, 4000, 10000$$

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- Check the speed of the computation of determinant of a matrix on various sizes of matrices A, produced by rand command as defined above. Determinant of a matrix is determined by the line command $>>$ `det(A)`.

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- Check the speed of the computation of determinant of a matrix on various sizes of matrices A, produced by rand command as defined above. Determinant of a matrix is determined by the line command $>>$ `det(A)`.
- Implement a similar test for the rank of a matrix, which is computed by the line command $>>$ `rank(A)`

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